

Exercises to 1.4 Indirect proof

(1) Indirectly prove that the following formulas are tautologies:

1) $\neg(p \vee q) \rightarrow \neg p$

2) $(p \rightarrow q) \rightarrow [(r \vee p) \rightarrow (r \vee q)]$

3) $[p \vee (q \wedge \neg r)] \rightarrow [(p \vee q) \wedge (r \rightarrow p)]$

4) $[(p \rightarrow q) \wedge (r \rightarrow s)] \rightarrow [(p \vee r) \rightarrow (q \vee s)]$

5) $\neg p \rightarrow \neg(p \wedge q)$

6) $\neg p \rightarrow [(p \rightarrow q) \vee q]$

7) $[p \wedge (q \vee r)] \rightarrow [(p \wedge q) \vee (p \wedge r)]$

8) $[(p \rightarrow q) \vee r] \rightarrow \neg[(p \wedge \neg q) \wedge \neg r]$

(2) Indirectly prove that the following formulas are contradictions:

1) $(\neg q \rightarrow \neg p) \wedge \neg(q \vee \neg p)$

2) $[(p \rightarrow q) \wedge (p \rightarrow r)] \wedge \neg[p \rightarrow (q \rightarrow r)]$

3) $\neg[\neg(p \vee q) \vee (\neg q \rightarrow p)]$

4) $[(p \vee \neg q) \wedge r] \wedge \neg[\neg(p \wedge r) \rightarrow (\neg q \wedge r)]$