

Exercises to 1.7 Quick test of logical inference

(1) Determine by the quick test whether the following inference schemes are logically valid:

$$1) \frac{(p \leftrightarrow p) \rightarrow q}{q}$$

$$6) \frac{[p \vee (q \wedge r)] \wedge (p \rightarrow r)}{r}$$

$$2) \frac{p \wedge q}{p \leftrightarrow (q \wedge r)}$$

$$7) \frac{\neg p \wedge \neg q \wedge s}{p \leftrightarrow [q \rightarrow (r \leftrightarrow s)]}$$

$$3) \frac{\neg(r \rightarrow \neg s) \wedge p}{r \leftrightarrow (p \rightarrow s)}$$

$$8) \frac{(p \wedge \neg q) \vee [(r \wedge s) \rightarrow p]}{\neg p \vee (q \rightarrow r)}$$

$$4) \frac{(p \vee q) \rightarrow [(r \vee s) \rightarrow t]}{(p \wedge q) \rightarrow (r \rightarrow t)}$$

$$9) \frac{(p \rightarrow \neg q) \vee r}{\neg r \rightarrow p}$$

$$5) \frac{(r \rightarrow q) \wedge (s \rightarrow r) \wedge [p \rightarrow (r \vee s)]}{p \rightarrow (q \vee r)}$$

$$10) \frac{\neg(p \vee q) \wedge r}{[(p \wedge r) \rightarrow \neg q] \vee (\neg p \wedge \neg r)}$$

(2) Prove by the quick test that the following arguments are logically valid:

- 1) If I start a new job, I will have to buy a car, and if I go to the sea this summer, I will spend half of my savings. But if I buy a car and spend half of my savings, I will have to live on 10 euros a day. However, I cannot live on 10 euros a day. So either I won't buy a car or I won't go to the sea.
- 2) If our representative runs for president, if he runs a positive campaign, there will be a runoff. If there is a runoff and he wins the election, he will not be reelected in four years. However, if he supports the death penalty, he will win the election and be reelected in four years. Therefore, if our representative runs for president, if he runs a positive campaign, he will not support the death penalty.
- 3) If the doctor injects the antibodies, the patient will have an allergic reaction, and if he has an allergic reaction, his kidney will stop functioning. But if the doctor does not inject the antibodies, the bacteria will spread to the bloodstream. The patient's kidney will stop functioning if the bacteria spreads to the bloodstream. If the kidney stops functioning, the patient will not survive until the morning. Therefore, the patient will certainly not survive until the morning.
- 4) On New Year's Eve, John drinks red wine. If he celebrates with friends, he drinks beer. Therefore, if John celebrates the New Year with friends, he drinks red wine and beer.
- 5) If Alice enrolls in Ancient Greek, she will also enroll in Latin. If she enrolls in Ancient Greek and Latin, she will enroll in logic. But if she enrolls in Ancient Greek, then if she enrolls in logic, she will enroll in mathematics. Therefore, if Alice enrolls in Ancient Greek, she will also enroll in mathematics.

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- 6) If an economic crisis begins or international conflicts arise, if the government fails to act or takes inadequate action, there will be neither economic growth nor political stability. If there is no economic growth or taxes are raised, there will be protests. Therefore, if an economic crisis begins, then if the government fails to act, there will be protests.
- 7) If Peter has met Mary, he would have told her the news if he knew it. But Peter met Mary and did not tell her the news. So he didn't know it.
- 8) If I go to the ball, I'll have to buy a tailcoat. But if I buy a tailcoat, I will not be able to pay my rent and repay my loan at the same time. If I don't pay the rent, I'll have to hide from the landlord, and I can't do that. Besides, I'll have to repay the loan. So, I can't go to the ball.