Exercises to *1.9 Logical transformations*

(1) *For each formula, determine whether it is in disjunctive normal form, in conjunctive normal form, in both, or in neither.*

1) $s \lor \neg q \lor s \lor \neg r$

2) $(\neg p \land r) \lor (q \land \neg q) \lor s$

3) $(r \lor \neg q) \land \neg p \land (q \lor \neg(r \lor q) \lor \neg s)$

4) $\neg q \lor \neg(p \land q) \lor (\neg r \land q)$

5) $\neg r \land q \land \neg s \land r$

6) $\neg q \lor (\neg r \land s \land p) \lor \neg r$

7) $q \land (r \lor \neg s \lor \neg p) \land \neg r$

8) $\neg(r \land q) \lor (\neg p \land q) \lor r$

9) $(p \land \neg q \land r) \lor p \lor \neg q \lor r$

10) $(p \land q) \lor (p \land \neg q) \lor \neg(p \land q) \lor (\neg p \land \neg q)$

(2) *Convert the following formulas to disjunctive normal form and simplify them as much as possible:*

1) $(p \land \neg p \land q) \lor [q \land (q \lor r)] \lor q$

2) $\neg q \to \{\neg p \land [q \lor (\neg p \land \neg q)]\}$

3) $\neg[\neg(q \to r) \lor (r \land q)] \lor \neg(p \to q)$

4) $\neg q \to \{(r \to q) \to \neg[\neg r \land (\neg q \lor \neg p)]\}$

5) $(p \leftrightarrow q) \land (q \leftrightarrow r)$

6) $(\neg p \to q) \land (\neg p \to \neg q)$

7) $\neg\{[p \land (p \to q)] \to (q \land \neg r)\} \to q$

8) $[(p \land \neg q) \lor \neg p] \to \neg[(r \lor q) \to \neg q]$

9) $s \leftrightarrow (q \land r)$

10) $(p \land q) \leftrightarrow (q \to r)$

11) $[(r \to \neg s) \to r] \land \{(r \land s) \lor [\neg r \to \neg(p \to q)]\}$

12) $(p \land \neg r \land p) \lor \{[(q \to q) \to (p \to q)] \to q\}$

13) $(p \leftrightarrow q) \to (r \leftrightarrow q)$

14) $[p \land \neg(s \to q)] \lor [(p \lor q) \land (s \lor p)] \lor (s \land \neg q)$

15) $[(p \to q) \to p] \leftrightarrow \neg p$

16) $[(p \to q) \to q] \land \neg(p \to \neg q)$

17) $\{\neg[r \lor \neg(s \to p)] \land (s \to \neg p)\} \to (r \land s)$

18) $p \to [q \leftrightarrow (r \land s)]$

19) $\neg(\neg r \to \neg\{q \lor \neg[p \lor \neg(q \lor r)]\})$

20) $p \leftrightarrow (r \leftrightarrow q)$

(3) *Determine by logical transformations if the following pairs of formulas are logically equivalent.*

1) $\neg(\neg p \to q)$
$\neg(\neg q \to p)$

2) $(p \to q) \to \neg p$
$\neg p \land \neg q$

3) $(p \to q) \lor (r \to q)$
$r \to (p \to q)$

4) $p \to (\neg q \land r)$
$(p \to \neg q) \land (p \to r)$

6) $p \to [(p \land \neg q) \to r]$
$r \lor q \lor \neg p$

7) $(p \land \neg q) \to r$
$\neg r \to (\neg p \lor q)$

8) $(p \land q) \to (r \land s)$
$[(p \lor q) \to s] \land [\neg r \to (\neg q \land \neg p)]$

9) $\neg p \land \neg q \land \neg r \land \neg s$
$\neg(\neg p \to s) \land \neg(\neg q \to r)$

5) $(p \lor q) \land \neg(p \land q)$

$p \leftrightarrow \neg q$

(4) *Determine by logical transformations which of the following five sentences are logically equivalent:*

1) John will take to drinking, if he gets fired or Mary leaves him.
2) John will take to drinking, if he gets fired, or he will take to drinking if Mary leaves him.
3) John will take to drinking, if he gets fired and Mary leaves him.
4) John will take to drinking if he gets fired, but he will also take to drinking if Mary leaves him.
5) If John gets fired, then if Mary leaves him, he will take to drinking.

(5) *Prove by logical transformations that the following formulas are tautologies.*[1]

1) $\neg(p \lor q) \rightarrow \neg p$
2) $(p \rightarrow q) \rightarrow [(r \lor p) \rightarrow (r \lor q)]$
3) $[p \lor (q \land \neg r)] \rightarrow [(p \lor q) \land (r \rightarrow p)]$
4) $[(p \rightarrow q) \land (r \rightarrow s)] \rightarrow [(p \lor r) \rightarrow (q \lor s)]$
5) $\neg p \rightarrow \neg(p \land q)$
6) $\neg p \rightarrow [(p \rightarrow q) \lor q]$
7) $[p \land (q \lor r)] \rightarrow [(p \land q) \lor (p \land r)]$
8) $[(p \rightarrow q) \lor r] \rightarrow \neg[(p \land \neg q) \land \neg r]$

[1] The formulas are the same as in exercise **(1)** from **1.4 Indirect proof.** You can compare the two methods.