Exercises to *4.1 Modal logic*

(1) *Symbolize the following sentences in the language of modal* ***propositional*** *logic.*

1) John will not necessarily come to the meeting.
2) Necessarily, everything is finite or infinite.
3) If you are obliged to vote, then you are allowed to vote.
4) Smoking is forbidden in the room but is allowed in the hallway.
5) This is not only possible; it is possible by necessity.
6) This is necessary, but it is not necessary by necessity.
7) If this happened once, it will happen in the future.
8) It is possible that everything is necessarily finite.
9) This is not necessarily necessary, but it is possible by necessity.
10) If it is permissible for this to be mandatory, it is mandatory for it to be permissible.

11) If this ever happens, it has always been a fact that it will.

12) If this happened, it will always be a fact that it did.
13) If this is necessary, it is a fact, and if it is a fact, it is possible.
14) If it has always been a fact that this will happen, then there will come a time when it has already happened.
15) This already happened, but there was a time when it has not happened yet.

(2) *Are the following pairs of expressions logically equivalent? If not, change one of the modal operators in the first expression so that they become equivalent.*

1) $\neg\Box\alpha \quad \Diamond\neg\alpha$
2) $\neg\Diamond\neg\alpha \quad \neg\neg\Box\alpha$
3) $\Box\neg\Box\Box\neg\alpha \quad \neg\Diamond\Box\neg\Box\alpha$
4) $\Box\neg\neg\Diamond\neg\alpha \quad \neg\Diamond\Box\alpha$
5) $\Box\neg\Box\Box\neg\alpha \quad \Box\Diamond\Diamond\alpha$

6) $\neg\Diamond\Box\Box\alpha \quad \Box\Diamond\Box\neg\alpha$

7) $\Diamond\Diamond\neg\Box\Box\neg\alpha \quad \Diamond\Diamond\Diamond\Diamond\alpha$
8) $\Diamond\Diamond\neg\Box\Box\neg\alpha \quad \neg\Box\Box\Box\neg\Box\alpha$
9) $\Diamond\Diamond\neg\Box\Box\neg\alpha \quad \Diamond\Diamond\Diamond\neg\Diamond\neg\alpha$
10) $\Diamond\Diamond\Box\Box\neg\alpha \quad \neg\Box\Box\Box\Diamond\alpha$

(3) *Prove with diagrams that the following schemes are logically valid with respect to semantic rule* ***(1)***.

1) $\Box\alpha \rightarrow \Diamond\alpha$
2) $\Diamond\Box\alpha \rightarrow \Diamond\alpha$
3) $[\Box(\alpha\rightarrow\beta) \wedge \Box\neg\beta] \rightarrow \neg\alpha$
4) $(\Diamond\alpha \wedge \Diamond\beta) \rightarrow \Diamond(\alpha\vee\beta)$
5) $[\Box(\alpha \vee \beta) \wedge \Box\neg\alpha] \rightarrow \Box\beta$
6) $\Diamond(\alpha \vee \beta) \rightarrow (\Diamond\alpha \vee \Diamond\beta)$
7) $\Box\alpha \rightarrow \Box\Box\Box\Box\alpha$
8) $\Diamond(\alpha \vee \beta) \rightarrow (\Diamond\Diamond\alpha \vee \Diamond\beta)$
9) $[\Box(\alpha \vee \beta) \wedge \Box(\alpha\rightarrow\gamma) \wedge \Box(\beta\rightarrow\gamma)] \rightarrow \Box\gamma$

10) $\neg\Diamond(\alpha \vee \beta) \rightarrow \Box(\neg\alpha\rightarrow \neg\beta)$

(4) *Prove with diagrams that the following schemes are logically valid (with respect to semantic rule **(2)**) without imposing any conditions on the accessibility relation.*

1) $\Box(\alpha\rightarrow\beta) \rightarrow (\Box\alpha\rightarrow\Box\beta)$

2) $(\Box\alpha \wedge \Box\beta) \rightarrow \Box(\alpha \wedge \beta)$

3) $\Box\alpha \rightarrow \Box(\alpha \vee \beta)$

4) $(\Box\alpha \wedge \Diamond\beta) \rightarrow \Diamond(\alpha \wedge \beta)$

5) $\Box(\beta\rightarrow\alpha) \rightarrow \neg\Diamond(\neg\alpha \wedge \beta)$

6) $\Box(\alpha\rightarrow\beta) \rightarrow (\Diamond\alpha\rightarrow\Diamond\beta)$

7) $\Box\Box\alpha \rightarrow \Box(\Diamond\alpha \vee \Box\beta)$

(5) *Prove with diagrams that the following schemes are logically valid when the accessibility relation is **reflexive**.*

1) $\alpha \rightarrow \Diamond\alpha$

2) $\Box\alpha \rightarrow \Diamond\Diamond\alpha$

3) $\Box(\Box\alpha\rightarrow\alpha)$

4) $\Box(\alpha\rightarrow\Diamond\alpha)$

5) $\Box\Box\Box\alpha \rightarrow \alpha$

6) $\Box(\alpha\rightarrow\beta) \rightarrow (\alpha\rightarrow\Diamond\beta)$

7) $[\Box(\neg\alpha\rightarrow\beta) \wedge (\alpha\rightarrow\Box\beta)] \rightarrow \Diamond\beta$

(6) *Prove with diagrams that the following schemes are logically valid when the accessibility relation is **transitive**.*

1) $\Diamond\Diamond\alpha \rightarrow \Diamond\alpha$

2) $\Diamond\Diamond\Diamond\alpha \rightarrow \Diamond\alpha$

3) $\Box\alpha \rightarrow \Box\neg\Diamond\Diamond\neg\alpha$

4) $[\Diamond\Diamond\Diamond(\alpha\rightarrow\beta) \wedge \Box\alpha] \rightarrow \Diamond\beta$

5) $\Box(\alpha\rightarrow\beta) \rightarrow \Box(\Box\alpha\rightarrow\Box\beta)$

6) $\Box\Diamond\alpha \rightarrow \Box\Diamond\Box\Diamond\alpha$

(7) *Prove with diagrams that the following schemes are logically valid when the accessibility relation is **symmetric.***

1) $\Diamond\Box\alpha \rightarrow \alpha$

2) $\Diamond\Box\alpha \rightarrow \Box\Diamond\alpha$

(8) *Prove with diagrams that the following schemes are logically valid when the accessibility relation is* ***symmetric*** *and* ***transitive****.*

1) $\Diamond\Box\alpha \rightarrow \Box\alpha$

2) $\Box(\Box\alpha \vee \beta) \rightarrow (\Box\alpha \vee \Box\beta)$

3) $\Diamond\Box\alpha \rightarrow \Box\Box\alpha$

(9) *Prove with diagrams that the following schemes are logically valid when the accessibility relation is* ***reflexive*** *and* ***transitive****.*

1) $\Diamond\Diamond\Box\alpha \rightarrow \Diamond\alpha$

2) $(\Box\alpha \vee \Box\Box\Box\beta) \rightarrow (\Box\Box\Box\alpha \vee \Box\beta)$[1]

[1] You should consider different cases in the proof.